Multireservoir Simulation Using Multipurpose Constraints and Object-Oriented Software Design

Marcelo A. Cicogna¹, Darrell G. Fontane², Ieda G. Hidalgo² and João E. Lopes³

- ¹ Anhanguera Educacional S.A., Alameda Maria Tereza, 2.000, Valinhos, São Paulo, Brasil; PH 55 (19) 3512-1715; email: marcelocicogna@gmail.com.
- ² Dept. of Civil Engineering, Colorado State University, Ft. Collins, CO 80523; email: {darrell.fontane, ieda.hidalgo}@colostate.edu.
- ³ Duke Energy International Geração Paranapanema, Av. Das Nações Unidas, 12.901, São Paulo, SP, Brasil; email: jelopes@dglnet.com.br

ABSTRACT

The hydroelectric short term planning is quite challenging in order to respect all the operational restrictions concerned with hydropower generation. A detailed simulation model combining multipurpose constraints and object-oriented software design is presented for short term decision support. The object-oriented design permits a mathematical structure that separates the decision tasks from the regular hydroelectric computation. The main operational policy allows users to evaluate a hydro generation schedule using inflow forecasting. In other words, the simulator can accept hydro generation as data input and convert it to water flow variables (turbines discharge and spillage), using the inflow forecast and the initial reservoir's storage. The water balance equation, hydro generation and efficiency calculation modules are synchronized with multireservoir operation. As an important result, this simulator can automatically correct the input data of the operational policy, saving substantially time in the decision-making process. A test problem on 94-reservoir subsystem within the Brazilian Integrated Hydropower Generation System was simulated using an hourly time-step over a one-week horizon with good performance, demonstrating the simulation model's capabilities for solving a large scale hydropower operation problem.

SIMULATION MODEL

In reservoir management practices, a simulation model can be used as a valuable planning tool to evaluate the impact of changes to the system's configuration or operational objectives. The desired generation or release scheduling can be checked using inflow forecasting in order to satisfy the entire set of operational constraints. At the real time operation stage, a simulation tool can be used to quickly check operational alternatives due to emergency events or planning and real-time incongruence. In post-operation studies, a simulator can be applied as a data consolidation tool in exhaustive inspection of recorded trajectories feasibility (Hidalgo et al., 2009).

Inserted in a computer-aided scenario, Yeh (1985) cites simulation as a modeling technique that is used to approximate the behavior of a system on the computer, representing all the characteristic of the system by a mathematical description. Wurbs (1993) provided extensive list of simulation computer models for reservoir system analysis. Using the classification presented by Wurbs, this model can be classified as a conventional simulation model in the sense that no formal optimization or mathematical programming algorithms are used. However,

it will be shown that the data structure for the joint operation of hydro plants can be represented as a network flow model and this design can represent a detailed hydroelectric mathematical model.

The simulation of multipurpose reservoir operation is also presented. Fontane (1997) provided a relevant reservoir operations planning model through fuzzy dynamic programming to deal with imprecise objectives. This model guides the multipurpose implementation of the present simulation model.

The mathematical model and software design are presented with emphasis on a practical application so that future researchers are able to reproduce these results. The authors believe that any kind of optimization model for reservoir operation has limitations (Labadie, 2004). Also, it is hard to find comparison studies among different models and their results. In this sense, different operational policies can be checked in a detailed simulation model, with a particular object-oriented design, to address performance analyses and to evaluate distance between model results and real problem aspects.

A test study on 94-reservoir sub-system of Brazilian hydro plants is presented as demonstration of simulation model's capabilities and computational performance. The user interface is described to present useful software resources for multireservoir operation analysis. The concluding section discusses design and implementation experience and plans for future development.

Reservoir Model

The reservoir operation has limits and constraints associated with the maximum and minimum water levels, as described in Figure 1. For reservoirs with hydroelectric powerhouses, the minimum level takes into account the minimum head that permits generation. The z^c level represents the dam crest elevation.



Figure 1. Reservoir operational water levels.

Above the maximum operational level, there is an absolute maximum water level related to dam security. The simulation model presented here has a particular computation module that automatically deals with reservoir constraints under emergency situations, as will be described into the software design section.

Hydroelectric Model

The simulation model has a detailed representation of the hydro plants, focused on a complete set of operation constraints and nonlinear equations. Figure 2 illustrates an arrangement of the main operational variables. This paper uses the

SI units standard to present the data for the Brazilian hydro plants, although the modeling resources can use other standards for general application.



Figure 2. Hydro plant's main variables arrangement.

Hydroelectric power availability is directly related to the potential energy created by a dam, i.e. it depends on the difference between upstream and downstream water levels. Therefore the gross head is directly calculated as:

$$h_g = z_{fb}(x) - z_{tr}(u) \tag{1}$$

where:

- h_g is the gross head (m).
- x is the water storage in the reservoir $(10^6 \text{ m}^3 \text{ or } \text{hm}^3)$.
- *u* is the total water release (outflow), that is, the sum of the discharge q and the spillage s (m³/s).
- $z_{fb}(x)$ is the forebay elevation (m) as a function of the water storage x.
- $z_{tr}(u)$ is the tailrace elevation (m) as a function of the water release u.

The forebay and tailrace equations could be expressed as any function of reservoir storage and total outflow, respectively. In the case of Brazilian hydroelectric plants, those functions are commonly represented by polynomials.

There are also losses from the potential energy due to friction of flow through raceways, racks and gateways (Mead, 1908). These losses can be estimated as a quadratic function of turbine discharges, as expressed in Equation (2). Therefore, the penstock head losses are measured in length units.

$$h_{pl} = c.q^2 \tag{2}$$

where:

 h_{pl} is the penstock head losses (m).

- c is a hydraulic loss parameter (s^2/m^5).
- q is the total turbine water discharge (m³/s).

After the head and losses description, the kernel of hydro plant operation is the hydropower production function. This model details the hydro production as shown in Equation (3).

$$p = k \cdot \eta_T \cdot \eta_G \cdot (h_g - h_{pl}) \cdot q \tag{3}$$

where:

- *p* is the instantaneous power obtained in the conversion process of the hydraulic potential energy to electric energy (MW).
- k is the gravity constant, multiplied by the water specific weight and divided by 10⁶. This constant manages the power output in MW. Its value is 0.00981 (MW/(m³/s)/m). This constant can also embed unit transformation factors.
- η_T is the efficiency parameter of the turbine in the conversion process of the potential energy to mechanical energy.
- η_G is the efficiency parameter of the generator in the conversion process of the mechanical energy to electrical energy.

The turbine efficiency can be expressed as function of head and hydro power. Some machinery projects have head and discharge as basic input data for efficiency computation. This simulation model accepts any combination from gross or net head and power or discharge. Due to its shape, this function is also known as the efficiency's *hill curve* and an example could be noted in Figure 3a. Also, efficiency contour lines are commonly part of the turbines information (Figure 3b). The turbine and generator manufacter has operational limits expressed as functions of gross or net water head. The maximum hydro power limit depends on head in two stages (Figure 3b): a) for head smaller than h^{ef} , the power is direct proportional to the head, since it is limited by hydraulic available energy; b) when head is bigger than h^{ef} , the power transferred to the turbinegenerator's shaft can damage the equipment, so valves or wicket gates gearing are necessary to keep the output power constant and beneath the electrical damage limit. The h^{ef} head is called *effective head*. A minimum hydro power limit can be imposed and expressed as a function of water head as well (Figure 3b).

Turbine Operation Characteristics (a) (b) = g(h) p^{ef} 0.9 Efficiency 0.90 0.87 0.82 Head Power (m) h^{\min} h^{\max} h^{ef} (MW)

Figure 3. Example of the turbine efficiency function - hill curve - and maximum and minimum power productions functions.

The maximum spill capacity s^{max} can be expressed as a function of reservoir water level z_{fb} . Depending on the existence of gates, two spillway operations are possible: controlled and uncontrolled. Figures 4a and 4b show examples of these functions. The term s^{prj} is related to the capacity projected for the spillway. The forebay z^{sc} is the water level on the spillway crest.



Figure 4. Uncontrolled and controlled maximum spill functions.

As can be seen, the simulation model has a detailed representation of the reservoir and hydro power operation. The next section describes the multireservoir model and the adopted implementation structure and design.

Multireservoir Simulation

This section presents a network flow formulation for the multireservoir simulation problem as a temporally expanded arborescence (a system with a treelike structure, Rosenthal, 1981). Figure 5 shows the hydro elements represented by the simulation model. The model assumes that the exogenous inflows y to the reservoir system are deterministic or provided by inflow forecasting models. Reservoirs could be simple water storage reservoirs such as node number 3, or could have powerhouses and hydro plants. Special river flow control stations can be simulated as well (node 7). Diversions structures, such as channels and tunnels, are able to be simulated, as illustrated by the linked-reservoir 3 and 4 example. The practical 94-reservoir study present later is a bigger replication of the hydro elements example shown in Figure 5. Also, this study simulates two real diversion structures from the Brazilian hydroelectric system: the Pereira Barreto's channel that links the Ilha Solteira and Três Irmãos reservoirs located in the Paraná River basin and a diversion tunnel linking the reservoirs Jordão and Segredo in the Iguacu River basin. These water diversions are governed by the hydraulic rules for the interconnecting channels and tunnels.



Figure 5. Hydro elements available for multireservoir simulation.

Rosenthal's representation, a deterministic network flow optimization model, requires that no reservoir has more than one other reservoir directly downstream from it. This constraint is valid for the 94-reservoir Brazilian hydropower system. However, the diversion variable was created to overcome this restriction, since a simulation model should accommodate structural expansions. The second imposition was related to the routing effect between adjacent reservoirs. This restriction was overcome with adaptations in the expanded arborescence structure, as shown in Figure 6.



Figure 6. Three-reservoir and four-interval hydro system as a temporally expanded arborescence with routing effect representation.

In the routing effect representation illustrated in Figure 6, solid arrows represent releases and broken-line arrows represent storages transported between time intervals. These storage arcs form the temporally expanded network structure from the arborescence. Note that two additional arc-node sets are necessary to complete the structure: the left represents the recent past releases before the first simulation time interval; and the right are the post-horizon arcs and nodes, without release arcs, that are necessary to complete and maintain mass balance.

The reservoir mass balance equation (4) is very detailed in terms of multipurpose constraints.

$$x_{i,t} = x_{i,t-1} + \left(y_{i,t} + \sum_{k \in \Omega_i} f(u_{k,t-tr}) - \left(q_{i,t} + s_{i,t} + ev_{i,t} + mu_{i,t} + d_{i,t} \right) \right) f_{uc}$$
(4)

where:

- *i*,*t* are plant and time indexes, respectively. There are *N* reservoirs and *T* intervals. The storage variables *x* are indexed as time instants (initial *t*-1 and final *t*). The flow variables are averages during the time interval.
- y is the reservoir inflow (m^3/s).
- Ω is the direct upstream reservoirs index set. With the *tr* time lag, the sum of upstream releases represents the routing effect computation.
- s is the spillage flow (m^3/s).
- *ev* is the reservoir evaporation calculated as a water flow (m³/s). It takes into account the reservoir area at the average volume $(x_t + x_{t-1})/2$, leading to an iterative calculus.

- mu represents the reservoir multiple-uses expressed as water flows (m³/s). As example, irrigation and water supply associated with one reservoir could be simulated with this variable.
 - *d* represents the water diversions from reservoir *i*, also expressed as water flow (m^3/s). The diversion can be reach another reservoir, river or an external hydro element not present into the study configuration.
- f_{uc} is the unit conversion factor between flow and storage variables.

As a powerful modeling resource, the unit conversion factor f_{uc} embeds into the simulation model the ability to use any time interval. This model can compute studies with monthly, daily, hourly, half hour or even any combination that composes a mixed interval horizon. Considering Brazilian general units hm³ for storage and m³/s for streamflows, this factor became:

$$f_{uc} = \frac{\Delta t}{10^6} \tag{5}$$

The multireservoir model includes the operational constraints related to storage, minimum and maximum releases, maximum discharge as a function of gross head and maximum spill as a function of reservoir storage level.

$$x_{i,t}^{\min} \le x_{i,t} \le x_{i,t}^{\max} \tag{6}$$

$$u_{i,t}^{\min} \le u_{i,t} \le u_{i,t}^{\max} \tag{7}$$

$$p_i^{\min}(h) \le p_{i,t} \le p_i^{\max}(h) \tag{8}$$

$$q_{i,i} \le q_{i,i}^{\max}(h) \tag{9}$$

$$S_{i,t} \le S_i^{\max}(z_{fb}) \tag{10}$$

It is important to note that constraints (6) to (10) use interval indexing to represent the multipurpose constraints, as presented in following section.

Multipurpose Constraints

Typically, reservoir multiple purposes include a combination of purposes like hydroelectric power generation, water supply for irrigation, domestic and industrial use, water quality improvement, flood control, wildlife and environment maintenance, navigation and recreation (Yeh and Becker, 1982).

The hydroelectric purpose was described in previous subsections. Through multiple-use *mu* and water diversion *d* variables from Equation (4) it is possible to simulate water supply and irrigation purposes. A seasonal minimum water level or storage may be imposed to guarantee water supply impounding, navigation and recreation purposes as well. The release constraints from Equation (7) permit simulating fish-wildlife maintenance and navigation operational limits. Flood control is generally achieved by maintaining reserve storage in each of the many reservoirs as an anticipation control procedure for high-flow periods. This can be achieved with temporary upper bound storage limits that are smaller than the reservoir maximum, as expressed in Equation (6).

SOFTWARE DESIGN

This section describes the use of object-oriented modeling as a powerful approach for designing a multireservoir simulator. Slobodan et al. (1997) present

object-oriented design as a successful modeling tool to the Egyptian water resources planning. The natural object-oriented partition between policy commitment and data permits the construction of "what-if" search scenarios in an easier way to facilitate the decision-making process. Another interesting objectoriented experience is detailed presented by Belkhouche (1999) and can be used as a guide to object-oriented design of database and software resources to water quality systems. Also, Horstmann (1997) provides an informative introduction to object-oriented software development.

The present simulation model was implemented using object-oriented software design. As expected, the software was built using classes to represent all multireservoir simulation aspects. There are classes to represent the hydro plant's main components and the network flow elements. Also, there are extra classes to support the software organization, such as horizon description and database connectors and data providers. The whole class diagram is quite lengthy, however, some important details from the object-oriented design are shown to guide further researchers.

Data Structures

The key step for a computational implementation of the simulation model is to interpret the mass balance equation (4) as a network flow node (Rosenthal, 1981). The node flow equilibrium has equivalence with the mass balance using arcs to represent operational constraints, as presented in Figure 7. The tr time delay represents the routing effect of upstream releases.



Figure 7. Mass balance equation equivalence with a network node.

The nodes for the entire network can be simply stored as a bi-dimensional matrix M_{NxT} representing the operation variables from the reservoir *i* during the time interval *t*. Other data structures using a node's linked lists can be used as well, but a rectangular matrix provides good performance and reasonable size storage. The arcs are built as links between nodes and stored as nodal properties.

Object-Oriented Design

The simulation model uses an object-oriented resource called *polymorphism* to create a mathematical structure that splits the decision tasks from the hydroelectric computing tasks in different software components. As a direct result, the simulation model accepts different operational policies, keeping apart the hydro computational kernel composed of the operational restrictions, water balance equation, power generation and efficiency functions. A simulation algorithm flowchart is shown in Figure 8 in order to illustrate the polymorphic composition of the operational policy and the hydro computational kernel. Note that the hydro kernel has a special resource to correct decisions that can lead the reservoir to an infeasible state.



Figure 8. Simulation algorithm with object-oriented polymorphism applied to partition of operational policy and the hydro computational kernel.

The polymorphism concept has many definitions but to this simulation model it means that *the same operation may behave differently for different classes* (Horstmann, 1997). In other words, the polymorphism will be used to promote a partition between the operation policy and the hydroelectric and reservoir computations. This object-oriented concept permits the resultant simulation software to have an expandable and adaptable collection of operation policies.

Operational Policies

The operation policy object is responsible to determine the reservoir's total water release. If the reservoir has a powerhouse, the amount of the turbines' discharge is also necessary. Mathematically, the operational policy can be expressed as a function f_p of the available policy variables from Equation (4):

$$[q_{i,t}, s_{i,t}] = f_p(x_{i,t-1}, x_{i,t}^*, y_{i,t}, \sum_{k \in \Omega_i} f(u_{k,t-tr}), ev_{i,t}^*, mu_{i,t}, d_{i,t}^*)$$
(11)

where the * denotes variables that are not directly available for policy analyses and may be estimated. For example, the final storage at interval *t* and evaporation can be estimated as a function of the release determined by the operational policy. Any external operational policy that has as its result the release and spillage variables can be simulated with this model.

From this polymorphic structure, a particular operational policy was implemented to transform hydro generation dispatch, composed of generation goals and turbines' unit commitment, into the discharge and spillage decision variables. The generation-release transformation is based on an iterative and detailed procedure over the hydro production equation (3), where the power is given and the release and spillage are calculated recursively. This policy aims to check generation scheduling using an inflow forecast, given the initial reservoir storage state and keeping the hydroelectric kernel intact. As one can see, this polymorphic design can accomplish other types of policies, including the decision results from optimization models (Wurbs, 1993).

CASE STUDY

This section briefly demonstrates a test on a 94-reservoir Brazilian hydro subsystem that demonstrates the simulator's capabilities and performance. The generation system has 75,944 MW as installed capacity and a one-week horizon generation scheduling was checked in terms of satisfying the hydroelectric constraints.



Simulation Study Main Characteristics

Reservoirs	94	
Basin/Rivers	8/33	
Drainage area	2,621,243	km ²
Installed capacity	75,944	MW
Diversion structures	2	
Control stations	2	
Horizon	7	days
Intervals	168	hours

Figure 9. The 94-reservoir map from Brazilian Integrated Hydropower Generation System and its main characteristics.

The test checks, for each reservoir, an hourly one-week horizon generation scheduling using a daily inflow forecast provided by an external hydrologic model. All the variables' trajectories (see section about multireservoir model) are available in singular tabs as charts or in tables. Figure 10 shows the software and user interface for a study's data input and result analyses. The left frame is a study navigator resource that shows studies in a tree view organization. The data and results access are provided by a tabs switch.

A HydroExpert					A HydroExpert							- 0 -×
Database Show Study Tools	Help			ľ	Database Show Study Tools	Help						
12 12 12	*	Demo SIN94	🔹 💣 🕺 🧶		12 12 12 · 13 · ·	1 0	😫 Demo S	JN94	- 6	K? 🧶		
HydroExpert	Cab	egory Model droSim HydroSim)	Study Demo SIN94	Status Simulated	HydroExpert	Cate	gory IroSim	Model HydroSimXI	•	Study Demo SIN94	Sta Sia	tus nulated
😑 🎦 HydroSim		Data Results		Reference date: 24/06/2006	🖶 🦳 HydroSim		Data Resul	5			Ref	erence date: 24/06/2006
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Scenario Policy Inflow Ad	vanced		HydroSimXP	8	System Per	r plant Grids				
		Per Plant Scheduling G	eneral Grid		CSU Demo 2 B 2 Demo CENIG B 2 Demo CENIG		Hydro elements		Flow Sp	lage Generation	Volume Units	Constraint + +
		Hydro elements	Decisions Charts Units	Validation R/R Dynamic		۲	- 21	Canoas II 🔺	Flow trajecto	/ trajectories		
		Ganoas II Canoas I	×		Demo ONS Single Composition	1	4	Capivara	17000	Water Nows - UHE Ital		
O Data	-	Capivara Taguarugu	Decisions grid - UHE Itaipu		O Data	-	Ś.	Rosana	15000			
⊕ ∰ Jupiá(37)-05-2007-		Rosana	Decisions Date H. Geo. OutBox	Units Mode *	8 Q Jupiá(37)-05-2007-		-4	Foz do Areia	12500	A A DA	10 0	A
B-92 Test Duke BDP 01 B-92 Test ONS BDT 01		Foz do Areiz	24/06 00-01 8584.4	0.0 1 14 20	Section 2 Construction 2 Constr			Segredo Santa Clara	분 10000-	PAR-14	NY	angen II
E Test ONS BDT 02		🗈 🛕 Segredo	01-02 8485.8	0.0 1 14 20	Test ONS BDT 02			Salto Santia	a 7500		J. N	V V
8-92 Test ONS BDT 03	100	A Santa Clara Salto Santia	02-03 8399.7	0.0 1 14 20	Test ONS BOT 03		2	Salto Osóric	5000			
E 4 Test RestrHidro 01		🗄 🍑 Salto Osóric	04:05 8468.9	0.0 1 14 20	E Test Restritidro 01		-ŏ:	Jauru				
		🕀 🔶 Salto Caxia	05-06 8515.7	0.0 1 14 20			-9	Itiquira I	2500-1	1 00000 00000 000	07 000 02 03	20,000,00,000,000
		Jauru Itinuira I	06-07 8552.6	0.0 1 14 20			X	Itiquira II	17	New Direct 17 Min (1 17 10	23 00030 00001
		Itiquira II	07-08 9649.7	0.0 1 16 20				Manso E		Outflow IV + Disch	aros	
		🕀 🥎 Ponte de Pe	08-09 9894.3	0.0 1 16 20			-Ā 1	Machadinho	-			
		🕀 🛕 Manso	09-10 9940.7	0.0 1 16 20			- 🔶 1	Itá				
<		A Machadinho	10-11 9901.0	0.0 1 16 20	<		- A	Passo Fund:				
Chi HydroData		K III F	11-12 9869.1	0.0 1 16 20	Ch HydroData			Ouebra Oue *				
Ch TermoData		94 Plant(e) 75944 44 MW	12-13 9813.3	0.0 1 16 20	Ch TermoData							
Q, HydroConsulta		Strangy Jost Hinn	10 10 10 0000 0	0.01 41461.00	Q, HydroConsulta		94 Mant(s)	75944.44 MW				
🖸 Convidado			Simulated	56.4% Version 1.3 R5	🖸 Convidado					Simulated	56.49	Version 1.3 R5

Figure 10. Simulation software and its user interface.

The test was run in computer with a 3.0 GHz Pentium IV processor and 2 GB of RAM. The 94-reservoir and 168-hour horizon study takes 2.4 seconds on average to be simulated. As one can note, this is good computational performance for a simulation model with the operational policy that converts generation goals into release and spillage trajectories with auto constraint feasibility checking.

CONCLUSION

This paper presents a multireservoir simulation system with detailed hydroelectric modeling and multipurpose constraints. The software is based on network flow structure to represent multireservoir joint operation. Two simple, but powerful, model resources were presented to permit general simulation application: a unit conversion factor for the hydropower production function and a multi and mixed time interval factor for the mass balance equation. Due to the object-oriented design, the operational policies and hydro computational kernel were divided into different software components. With this design it is possible to check different and external operational policies in a detailed simulation model. Also is suitable to address performance analyses and evaluating the distance between computer-aided model results and real problem performance.

A test study on 94-reservoir subsystem of Brazilian hydro plants with a one-week horizon generation scheduling was made using inflow forecasting with a focus on meeting the operational constraints. The simulation model performed very well in terms of reservoir operation accuracy and required computational time.

REFERENCES

- Belkhouche, B., Demtchouk, I., and Steinberg, L. J. (1999). "Design of objectoriented water quality software system." J. Water Resour. Plan. Manage., ASCE, 125(5), 289-296.
- Fontane, D. G., Gates, T. K., and Moncada E. (1997). "Planning reservoir operations with imprecise objectives." J. Water Resour. Plan. Manage., ASCE, 123(3) 154-162.
- Hidalgo, I. G., Fontane, D. G., Soares Filho, S., and Cicogna, M. A. (2009). "A simulator of the hydroelectric plants operation as a tool for analyzing data." Accepted for the 2009 World Congress on Computer Science and Information Engineering (CSIE), Los Angeles/Anaheim, California, USA.
- Horstmann C. S. (1997). *Practical Object-Oriented Development in C++ and Java*. John Wiley & Sons.
- Labadie J. W. (2004). "Optimal operation of multireservoir systems: state-of-theart review". J. Water Resour. Plan. Manage., ASCE, 130(2), 93-111.
- Mead, D. W. (1908). "Chapter II Hydraulics." *Water Power Engineering: The theory, investigation and development of water powers*, First Edition, McGraw-Hill, New York, 42-78.
- Rosenthal, R. (1981). "A nonlinear network flow algorithm for maximization of benefits in a hydroelectric power system." *Operation Research*. 29(4), 763-786.
- Sigvaldason, O. T. (1976). "A simulation model for operating a multipurpose multireservoir system." *Water Resources Research*, 12(2), 263-278.
- Slobodan, P. S., Hussam, F., and Amin, E.-S. (1997). "The use of object-oriented modeling for water resources planning in Egypt." *Water Resources Management*, Kluwer Academic Publishers, 11, 243–261.
- Yeh, W. W-G., and Becker L. (1982). "Multiobjective analysis of multireservoir operations." *Water Resources Research*, 18(5), 1326-1336.
- Yeh, W. W-G. (1985). "Reservoir management and operations models: A state-of-the-art review." *Water Resources Research*, 21(12), 1797-1818.
- Wurbs, R.A. (1993). "Reservoir-system simulation and optimization models." J. Water Resour. Plan. Manage., ASCE, 119(4), 455-472.